# Chapter 5
# Networking of Theories in the Tradition of TME

**Angelika Bikner-Ahsbahs**

## 5.1 The Networking of Theories Approach

During the last decade, the discussion on theory development has been reconsidered, for example, by Prediger (2010), in the networking of theories approach worked out by a group of European researchers[1] coming from Germany, Italy, Spain, Israel, France, and (at the beginning also from) the UK. The growing complexity of the field and an increase of the diversity of theories motivated this work (see Dreyfus 2009). The aim was to find a scientifically based way of dealing with different theories in research. The current state of the art on the networking of theories approach has been published in a recent book, illustrated by empirical case studies (Bikner-Ahsbahs et al. 2014), and in methodological articles (Kidron and Bikner-Ahsbahs 2012; Bikner-Ahsbahs and Prediger 2010; Prediger et al. 2008; Dreyfus 2009).

The idea of the networking of theories is based on four assumptions (Bikner-Ahsbahs 2009):

1. Regarding the diversity of theories as a form of scientific richness,
2. Acknowledging the specificity of theories,
3. Looking for the connectivity of theories and research results,
4. Developing theory and theory use to inform practice. (p. 7).

[1]Members were and still are: Angelika Bikner-Ahsbahs (speaker), Michèle Artigue, Ferdinando Arzarello, Marianna Bosch, Agnès Corblin-Lenfant, Tommy Dreyfus, Josep Gascón, Ivy Kidron, Stefan Halverscheid, Mariam Haspekian, Susanne Prediger, Kenneth Ruthven, Cristina Sabena, and Ingolf Schäfer.

A. Bikner-Ahsbahs (✉)
Faculty of Mathematics and Information Technology, University of Bremen,
Bibliothekstrasse 1, 28359 Bremen, Germany
e-mail: bikner@math.uni-bremen.de



A. Bikner-Ahsbahs et al., *Theories in and of Mathematics Education*,
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The networking of theories allows for explicitly working with different theories in order to benefit from their theoretical strengths with a specific focus on informing practice as well as being inspired by empirical situations of practice. To network theories means to build relations among theories. This approach is not a new idea. There are forerunners: for example, in 1992, Bauersfeld presented an integrated analysis of a teaching and learning situation using various theoretical approaches (1992b). He has also used strategies to compare and contrast radical constructivism and activity theory in order to clarify their (in-) compatibility (1992a). In 1998, Maier and Steinbring published a comparison of two theoretical approaches on processes of understanding based on the same empirical episode. These examples show that German researchers used the strategies of comparing theoretical approaches and integrating theoretical aspects to comprehend practice. Such strategies, called *networking strategies*, have been systematized, resulting in a landscape of four pairs of complementary strategies (Fig. 5.1). This landscape orders these strategies according to their potential for integration between two poles, non-relation of ignoring other theories, and unification of theories globally.

The first two pairs (understanding and making understandable, comparing and contrasting), acknowledge the theories' identities and the diversity of the theories as a resource in the field. They point to the basic necessity of understanding theories. This first pair may take place in a deepened way while comparing and contrasting theories. The second pair, comparing and contrasting, leads to awareness of differences and commonalities, thus learning from other perspectives. The third pair, combining and coordinating, is a step towards bridging theories. Its strategies allow working with different theories to build a conceptual framework or include complementary views into researching a problem. The fourth pair, locally integrating and synthesizing, leads to more comprehensive theoretical frameworks. Local integration may occur when a concept can be interpreted from different theoretical views, thus integrating the concept into other theories. Synthesizing is meant when two or more theories can be imbedded into a more holistic theoretical framework. While there are some cases of local integration, a case of synthesizing has not yet been achieved (see Bikner-Ahsbahs and Prediger 2010).

During the last decade, research methods using networking of theories have been developed in a number of projects. These methods encompass repeated exchanges

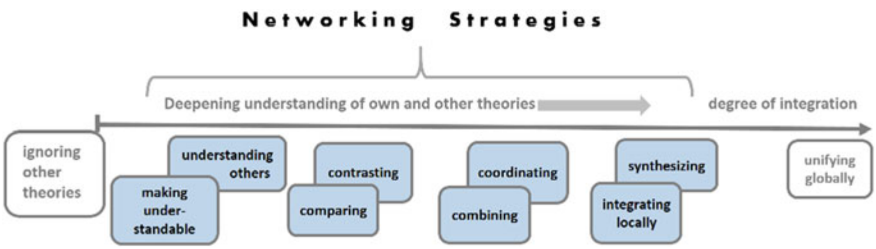


**Fig. 5.1** Networking strategies (Prediger et al. 2008, p. 170; Bikner-Ahsbahs and Prediger 2010, p. 492)

of experiments or analyses from various views, using networking strategies to connect theories, establishing a common ground, doing complementary analyses, implementing inclusive methodologies, and producing common methodological and theoretical reflections (Bikner-Ahsbahs 2009; see Bikner-Ahsbahs and Kidron 2015; Artigue and Mariotti 2014).

In contrast to TME, the foundation of a scientific discipline is not directly addressed in the networking of theories approach. Contribution to such a foundation is done by research connecting theories to solve problems and by additional meta-research of final reflections on methodologies and research results. For example, in Kidron et al. (2014), the notions of context were compared and contrasted in the theories "Abstraction in Context" (AiC), "Theory of Didactic Situations" (TDS), and "Anthropological Theory of the Didactic" (ATD). Since all three theories share an epistemological sensitivity, the comparison of the three relevant concepts, *context* in AiC, *milieu* in TDS and *dialectic media-milieu* in ATD, was related to the epistemological nature of the theories and made it possible to sharpen the notions of the three contextual concepts by comparing and contrasting their role in the theories. More generally, this case study showed three ways in which such a broad concept as "context" can be theoretically specified: as a core concept in TDS, as a developmental concept to answer the question of how media and milieu are interrelated in ATD, and as a variable counterpart of the theoretical core of abstraction processes in AiC.

Empirical research within the networking approach has also led to new kinds of theoretical concepts lying at the boundary of theories. These boundary concepts (see Akkermann and Bakker 2011) make sense from different theoretical views, mainly in complementary ways. For example, the "epistemological gap" (Sabena et al. 2014)[2] is a phenomenon that may appear when the epistemic view of the teacher and that of the students differ in that the students do not have access to the same epistemological resources as the teacher. In their example, a student explored the graph of the exponential function and how it develops for big x towards infinity. Using an asymptotic gesture, he described the way the slope of the graph increases towards infinity. His resource consisted of observing the graph on the computer screen; the teacher, however, asked for a proof of contradiction for the statement that the graph of the function could not have an asymptote. The student's epistemological view was based on perception and the teacher's epistemological view was based on logical arguments. They shaped an epistemological gap that the student could not bridge by himself. Two theoretical concepts were used in the analysis, the *semiotic bundle concept* and the concept of *interest-dense situation*. Both concepts integrated into a common methodology allowed for characterising and identifying this epistemological gap (Sabena et al. 2014).

[2]From a theoretical point of view, Burscheid and Struve (1988) have already described such an epistemological (teaching-learning) gap as the gap between the empirical theory of the students and a more logical theoretical background of the teacher.

Networking of theories has been developed as a research practice to solve problems that are so complex that more than one theory should be considered. It tries to handle the complexity of teaching and learning in research and to avoid simplistic interpretations. In contrast to TME, the networking of theories builds on concrete research taking into account different theoretical perspectives, reflecting the methodological processes and the epistemological basis.

Advancing the field is not an explicit issue of the networking of theories, but it can be a result of research conducted through a networking of theories approach. In this way, advancing the field does not happen in big steps but as a very slow process, layer by layer, based on research in the very same research process. As Artigue and Bosch (2014) have outlined, the meta-knowledge gained has not yet reached the level of theoretical or meta-theoretical knowledge so far; it is more a kind of craft knowledge enriched by methodological strategies. These strategies encompass meta-research, which clarifies theories and their assumptions, phenomena and concepts. Thus, the networking of theories further develops theories and the scientific dialogues that take place between researchers from distinct theory cultures (Kidron and Monaghan 2012).

## 5.2 The Networking of Theories and the Philosophy of the TME Program

The networking of theories was already foreshadowed in a discussion in the Topic Area on TME of ICME 5, just before the first TME conference, summarized by Steiner (1986):

> The ensuing discussion was basically concerned with a comparison of theoretical positions represented by two contributions concentrating on commonalities and differences between "information" and "knowledge."… In general, it was agreed that confronting and comparing different methods for interpretation and analysis of phenomena and problems in mathematics education is a worthwhile task and one to be worked at more intensely in future activities of TME. (p. 296)

Moving to the present day, the networking of theories approach has gone much further. It has developed strategies of *meta-research* building on the research itself as an additional research practice. Such strategies do not observe the field to identify basic problems to be addressed, and they do not offer big lines of theory development, but they do add a deep methodological reflection addressing complexity in the research practice. The new knowledge that has been produced consists of tiny but sustainable steps. In line with Bigalke and Steiner, it respects the *diversity of theories* as richness in the field. Thus, the disciplinary matrix of Kuhn and Masterman and specifically their developmental phases are not considered to be a suitable model for the field. Contextual and hence theoretical diversity, in the sense of networking of theories, provides rich insight mainly into research practices. The networking of theories approach may further develop theories and theory concepts and in this way advance the field. A *unifying paradigm* is explicitly excluded. Since this approach is a methodological and practical one leading to new kinds of concepts at the boundary of theories but also to new kinds of questions addressing complementarity, the advancement of the field may be reached through dialogue (Kidron and Monagham 2012). Research is not restricted to *home-grown theories* but research by networking of theories may develop the field in a *home-grown way*.

What is the potential to advance the field in the sense of TME if we practice networking of theories as a normal research practice? The reader is invited to engage in a networking of theories case and reflect on issues of TME. The example case will be on learning fractions. It will be analysed from the two theoretical perspectives presented before. The two analyses will then be networked to clarify the complementary nature of the two theories.

## 5.3 An Example of Networking the Two Theoretical Approaches

In a sixth grade class (partly presented in Bikner-Ahsbahs 2005, pp. 234–243, see Bikner-Ahsbahs 2001) the teacher implemented the following task to introduce the concept of fractions for the first time, giving the students three equal bars of chocolate represented as rectangles:

> Four students want to distribute three equal bars of the same chocolate in a fair way among them. How do they manage it? Find at least one distribution.

The students were supposed to work in groups and present their solutions to the class afterwards. Some of the distributions are shown in Figs. 5.2, 5.3, 5.4, 5.5, 5.6 and 5.7 (each student is represented by a different pattern).

In the class, a discussion about sameness and fairness took place. For example: Does everyone get the same in the distribution of Fig. 5.4 even though one gets three small pieces while the others get only one bigger piece each? In a similar way, sameness was discussed for the distributions in Figs. 5.5 and 5.7. The first implicit

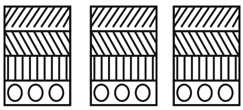

**Fig. 5.2** Distribution 1

**Fig. 5.3** Distribution 2

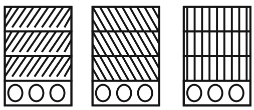

**Fig. 5.4** Distribution 3

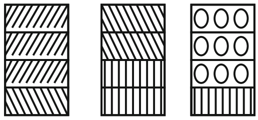

**Fig. 5.5** Distribution 4

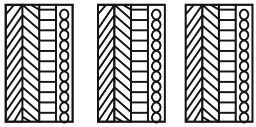

**Fig. 5.6** Distribution 5

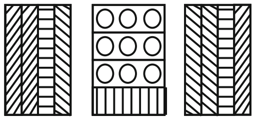

**Fig. 5.7** Distribution 6

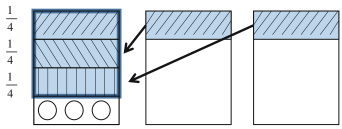


**Fig. 5.8** Substituting to check sameness

rule appeared to be: The pieces are the same when they can be substituted by the others. This was shown by the teacher in the diagram in Fig. 5.8 on the blackboard.

However, in Fig. 5.7 this was difficult to achieve. So the rule was changed to: The pieces are the same when they represent the same amount of chocolate. So why did the piece at the bottom of the second rectangle in Fig. 5.7 show the same amount of chocolate as the long parts in the first and the third rectangle? The answer was quickly found: one quarter of the same bars were always of the same amount no matter what shape the quarters have and how they are positioned. But now another question arose: What does everyone get? Three quarters of one bar? In Fig. 5.4, this seemed right for the parts with stripes, but not for the parts with circles. The latter parts rather were described as “three quarters of three bars,” while other students said that they were “one quarter of three bars.” This again caused a lively discussion about the question: Are three quarters of one bar the same as one quarter of three bars (and three quarters of three bars)? The subsequent discussion showed emotional engagement. Those students who interpreted the preposition *of* as *taken away from* were convinced that three quarters of one bar was not the same as one quarter of three bars because if just one quarter was taken, it was much less than three quarters. The whole as a variable entity was not yet built. One quarter or three quarters were regarded as pieces of an absolute size and not of a relative size

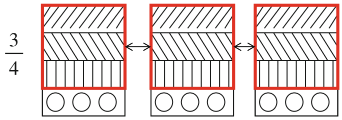


**Fig. 5.9** Three quarters of one big bar (three small bars)

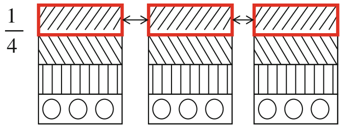


**Fig. 5.10** One quarter of one big bar (three small bars)

according to the related entity. Rosa had a nice idea about changing the size of the bar (Bikner-Ahsbahs 2005):

Rosa If we now, if we now join all the three bars together and then we would take from them three *quarters* [emphasized], that would be *too much* [emphasized]. This does not work if one would get three quarters of three bars. (p. 242, translated)

She joined the three bars, getting one big bar (represented by double arrows). Three quarters of this big bar would then be much more than just one quarter of the big bar (Figs. 5.9 and 5.10).

Thus, it became clear that in Fig. 5.4 the part with circles is one quarter of a big bar and that this was the same as three quarters of a small bar, still considering it to be a *part that is taken away*. This was still not acceptable for those students who regarded one quarter as an absolute size. One quarter as a relation between the part and the whole needed further exploration with variable entities, for example varying the size of the whole and investigating what *one quarter of* means.

## 5.4 The Sign-Game View[3]

The task has initiated an activity by setting the rule to achieve a fair distribution of the chocolate bars represented in the rectangles to be used. The students invented diagrams of distributions showing "the spatial relationships of its parts to one another and the operations and transformations of and with the diagrams" (Dörfler, Sect. 4.3) and inventing the rule that being the same means to be able to substitute the parts (Fig. 5.8). Based on the rule, they used "inventive and constructive manipulation of diagrams to investigate their properties and relationships." (ibid., Sect. 4.3). The students compared their solutions and tried to understand the diagram in a social activity, expressing their interpretations "in natural language and specific terms relating to the diagram," (ibid., Sect. 4.3) such as one quarter or three quarters

[3]We will use quotes to refer to Dörfler's text of Chap. 4 in this book.

in the different figures. “These descriptions and explanations cannot be substituted for the diagram and its various uses, however. In relation to the diagram and its intended relations and operations, this [language] is a meta-language about the diagrams, which also focuses attention and interest on its relevant aspects and activities.” (ibid., Sect. 4.3) These aspects and activities consist of the various ways in which three quarters are expressed by diagrams and what they mean compared to each other. However, it also shows that language may result in difficulties; for example, in the question, “Are three quarters of one bar the same as one quarter of three bars and three quarters of three bars?” While the diagrams seemed to be clear, the natural language of the students was not yet conventionalized; hence, the difficulty arose from the differences in the interpretation of what *quarter of/from* means. Exactly this aspect points to another difficulty the students had: regarding one or three quarters as a relationship between the part and the whole. The diagrams presented above do not show this aspect to be relevant. Rosa seemed to be aware of this relationship and invented a way of working with the diagram by changing the size of the whole bar. She began to build the whole as a variable entity by pushing the three bars together to achieve one big bar (Figs. 5.9 and 5.10). It is this action on the diagrams that shows what one quarter of a big bar means, and according to the original rule this is the same as three quarters of one small bar (Figs. 5.9 and 5.10). However, another rule must be added or disclosed by the students: one quarter or three quarters do not have an absolute size but must be used with reference to the whole. Rosa’s action shows that “the signs (see Figs. 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9 and 5.10) are not just a means or a tool for mathematical activity and creativity, but they are essential and constitutive for mathematics, its notions, and propositions and their meanings” and “sign activity can be executed with others and shown to others in a public form.” (ibid., Sect. 4.5). The students are engaged in a “sign game” with the help of the teachers accepting and inventing rules; thus developing mathematical meaning represented in processes of diagramming.

## 5.5 The Learning Activity View[4]

An activity theory view looks at the students’ actions while working on the task. Analyses are normally imbedded in the students’ learning biography in school focusing on the current situation and the teacher’s intentions and actions. A global view of planning the course of instruction is as relevant as the local view of the ways the teacher supports students in orienting and conducting the task. When choosing the task above, the teacher has to be aware of the cultural historical content which the task allows students to learn by the initiated learning actions. The question to be answered is what knowledge is a prerequisite and, hence, is or should be available.

[4]I thank Regina Bruder and Oliver Schmitt for their assistance in the analysis of the example.

In the given task, the students are supposed to learn the concept of a fraction, which is represented by figurative diagrams. While preparing this task, the teacher should insure that the necessary knowledge is available for carrying out the task, e.g., by implementing calculations for the area of rectangles. The task is supposed to initiate a learning action with the goal of finding out which equal parts of the chocolate bars the four people may get. As a resource, material chocolate bars may be offered and then transformed into iconic representations. This transformation might be introduced by the teacher as a helpful tool allowing mental ways of trying out and manipulating and finally transferring the results back to the real situation again.

Two basic acquisition actions are aimed at *identifying* and *realizing* fractions. First, the students begin to realize the fraction ¾ with the help of diagrams and identify other representations while comparing the students' solutions. The teacher systematizes the students' explorations during the class discourse to assist them in building a pattern or even field orientation for working with similar tasks. This is shown in the discussion about how far three single quarters of the chocolate bar correspond to one piece of ¾ of a bar. The teacher mediates between the knowledge the students have in mind and the knowledge that has been culturally given.

For further instruction, the teacher could use tasks for identifying and realizing fractions in terms of rectangular things or diagrams similar to the ones used before. Solving similar tasks with diagrams of a different shape such as a circle need not be successful based on pattern orientation, but might be a starting point for generalizing the knowledge about representing fractions.

## 5.6 Comparison of Both Approaches

The sign use approach built on Peirce and Wittgenstein focuses on the slow development of mathematical meaning being situated in manipulating diagrams, its perceivable changes, and diagrammatic reasoning. "This is very different from imagining math as a kind of abstract and mental activity." (Dörfler, in this survey, Sect. 4.5). Although people speak about these diagrams, the mathematical ideas are expressed in the diagrams and not built by mental constructions or images of people. The strength of this approach is its sensitivity towards which diagrams and their development can express mathematical ideas in certain rule-based ways. In the situation above, the part-whole relation of a fraction is diagrammatically unclear since there is only one kind of entity representing the whole. Diagrams and acting with diagrams belong to the kernel of this theoretical approach. Intended applications may consider what kind of acting on diagrams can express which specific mathematical meanings, how language about diagrams is used, and, hence, how sign games can be shaped by people and social groups.

The theory of *learning activity* is also based on acting, but not with a focus on *diagrams to be acted on* but more on the subjects *who are acting on the diagrams as resources to achieve specific cultural knowledge*. Two basic actions are distinguished, identifying and realizing, which can be initiated by tasks. Taking pattern or field orientations into account allows for foreseeing what kinds of tasks the students might solve successfully. Initiating a learning activity does not only focus on the current task situation but requires also taking past learning experience and future goals into account. Tools, e.g., diagrams, do not belong to the kernel of the theory. Its kernel encompasses the concept of activity and how a learning activity can be shaped, initiated by tasks, and created by the learner with the help of the teacher. The teacher's role is crucial. Referring to the example above, one intended application is concerned with the problem of which further tasks the teacher can choose in order to assist the students in building the concept of ¾ to be represented by various shapes. The strength of this approach is its prescriptive nature for initiating learning activities, while diagrams may serve one kind of resource among others.

While both approaches share the sensitivity towards acting, the core concepts (e.g., diagram) of the one theory lie more in the periphery of the other (e.g., as a resources for a learning activity). If we take a networking of theories view and coordinate the analyses by using the two theoretical views, the empirical situation presented may be investigated according to two complementary questions: (1) what and how can acting with diagrams express mathematical ideas and (2) how can a task with certain goals be designed to initiate basic actions, such as identifying and realizing in a specific stage of the course of instruction, that are built on prior knowledge and preparing future goals to achieve cultural knowledge. Thus, both approaches complement each other and may enrich each other to inform practice (see TME program): coming from the learning activity we may zoom into (see Jungwirth 2009 cited by Prediger et al. 2009, p. 1532) diagram use, and coming from diagram use we may zoom out (ibid., p. 1532) to embed the diagram use into the whole course of the learning activity.